Nonminimum-Phase Optimum-Amplitude Bandpass Waveguide Filters

ALI E. ATIA, MEMBER, IEEE, AND ALBERT E. WILLIAMS, MEMBER, IEEE

***Abstract*—The synthesis of narrow-bandpass waveguide filters having flat group delay and optimum-amplitude characteristics is described. The design utilizes either single- or dual-mode waveguide cavities. Experimental results for a 40-MHz-bandwidth filter centered at 4 GHz show excellent agreement with theory.**

INTRODUCTION

HIGH-QUALITY microwave communications system applications require narrow-bandpass filters possessing good frequency selectivity, linear phase, and small in-band insertion loss. Normally, minimum-phase direct-coupled resonator filters [1] followed by group delay equalizers [2] (phase correctors) are employed. Although direct-coupled resonator filters are relatively simple structures, their insertion loss functions are restricted to all-pole functions (e.g., Butterworth or Chebyshev). Recently, it was shown [3] that optimum-amplitude waveguide bandpass filters whose insertion loss functions have ripples in the passband and real finite zeros of transmission in the stopband can be constructed by using dual-mode multiple coupled cavities. Nevertheless, these filters still require separate group delay equalizers.

Since it is known that cascading a nonminimum-phase network with an all-pass network results in a network of a higher degree than is actually necessary [4], the direct realization of a general nonminimum-phase transfer function would offer considerable advantages.

This paper presents practical waveguide filter structures capable of producing an optimum response. The approximation problem is not tackled, since, from the practical point of view, existing analytical solutions [4]–[6] or numerical procedures [7] can be efficiently used to generate solutions on a digital computer. It is demonstrated that the most general form of bandpass transfer functions of symmetrical networks can be realized in either single- or dual-mode coupled-waveguide cavities. The theory is applied to the realization of a twelfth-order nonminimum-phase and optimum-amplitude transfer function in a square-cavity dual-mode waveguide structure. The experimental results included are shown to agree closely with theory. Finally, the response of this filter is compared with that of a twelfth-order Chebyshev filter, an augmented linear phase filter, and an arbitrarily prescribed phase filter with monotonic stopband response.

Manuscript received May 14, 1973; revised October 22, 1973. This paper is based upon work performed in COMSAT Laboratories under the sponsorship of the International Telecommunications Satellite Organization (INTELSAT). Views expressed are not necessarily those of INTELSAT.

The authors are with the Transponders Department, RF Transmission Laboratory, COMSAT Laboratories, Clarksburg, Md. 20734.

THEORY

Fig. 1 shows an equivalent circuit of n narrow-band synchronously tuned cavities coupled in an arbitrary fashion. The couplings M_{ij} between the ith and jth cavities are assumed to be frequency invariant, which is a valid assumption for narrow bandwidths. The circuit is completely described by the coupling matrix M, which is a real $n \times n$ symmetrical zero-diagonal matrix whose (i,j) entry is the value of coupling M_{ij}. When this circuit is considered as a 2-port resistively terminated at the output port by the load R_2 and driven at the input port by a source of open-circuit voltage E and internal resistance R_1, it can be shown [8] that the normalized low-pass voltage insertion loss ratio $t(S)$, defined as

$$t(S) = 2\left(\frac{R_1}{R_2}\right)^{1/2} \frac{V_2}{E} \tag{1}$$

can be expressed in the form

$$t(S) = c\,\frac{P(S)}{Q(S)} \tag{2}$$

where

c	constant;
P and Q[1]	monic polynomials in S;
$\deg P$	$\leq \deg Q - 2$

and

$$S = j\lambda = j\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{3}$$

where ω_0 is the resonant frequency of all the cavities. Conversely, given $t(S)$ satisfying the aforementioned condition, and

$$|t(j\lambda)|^2 \leq 1, \qquad -\infty < \lambda < \infty \tag{4}$$

it can be shown that there exists a network of the form shown in Fig. 1 that possesses this given $t(S)$.

When even-order symmetrical networks are considered, a canonical form, shown in Fig. 2, always exists for a network of order $n = 2m$. The couplings $M_{i,i+1}$, $i = 1,2,\cdots,m-1$, all have the same signs, but the couplings

[1] Q is a strict Hurwitz polynomial.

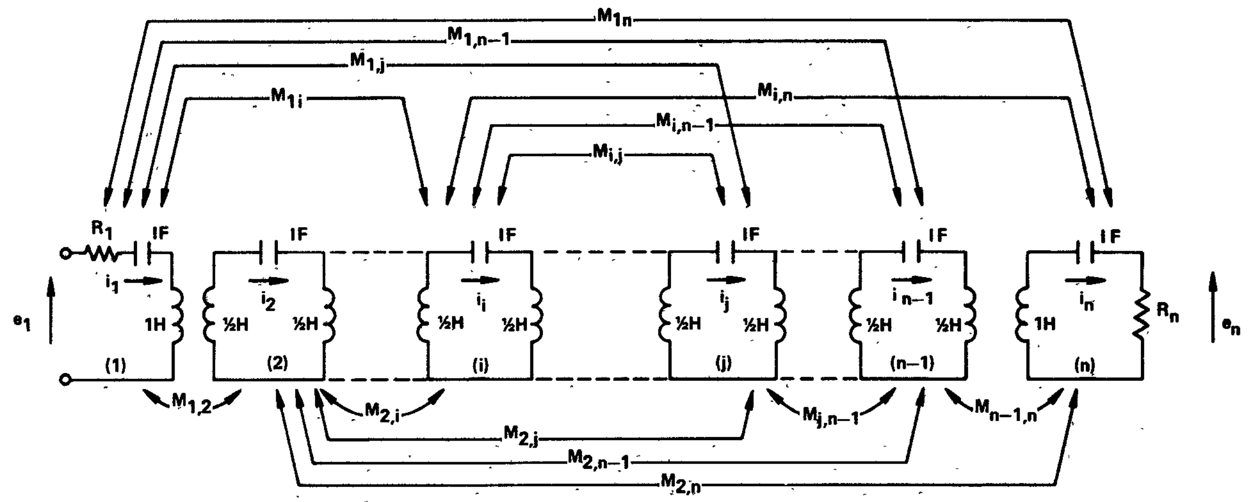

Fig. 1. General equivalent circuit of n arbitrarily coupled cavities.

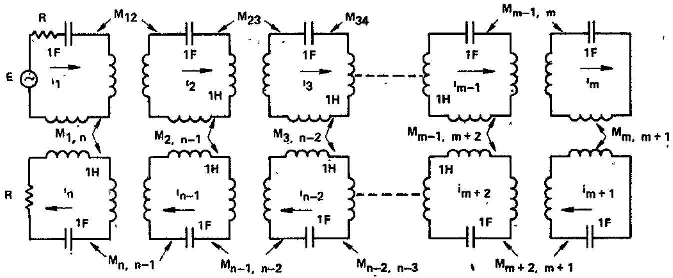

Fig. 2. Canonical form of equivalent circuit for symmetrical network of order $n = 2m$.

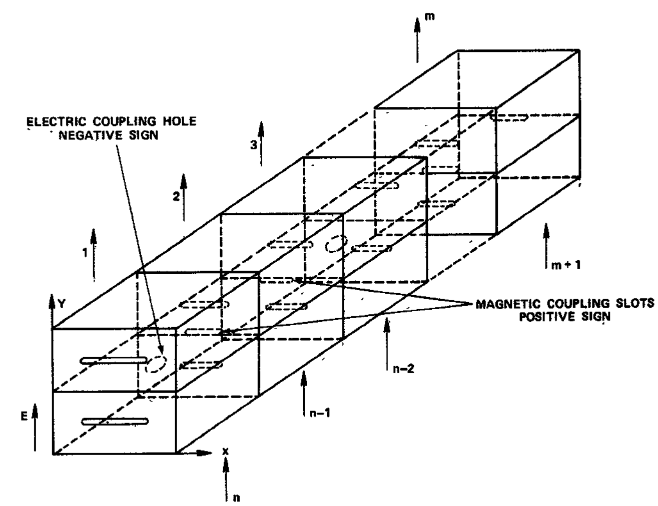

Fig. 3. Waveguide structure that realizes canonical form of symmetrical network.

$M_{i,n-i+1}$, $i = 1,2,\cdots,m$, can generally have different signs. This canonical form can be characterized by a unique "even-mode" $m \times m$ symmetrical-coupling matrix M_e, which is tridiagonal with diagonal elements $M_{e_{ii}} = M_{i,n-i+1}$ and off-diagonal elements $M_{e_{i,i+1}} = M_{i,i+1}$, for $i = 1,2,\cdots,m-1$. The presence of the diagonal-coupling elements $M_{e_{ii}}$ and the arbitrariness of their signs prevent the realization of the canonical network shown in Fig. 2 in any of the waveguide structures previously described [1], [6], since these can generate only couplings of the same sign. On the other hand, the waveguide structure [9] shown in Fig. 3 is a general realization of the canonical form of the equivalent circuit shown in Fig. 2. Each of the rectangular-waveguide cavities in Fig. 3 supports only its fundamental TE_{101} mode and resonates at a common center frequency. The difference between this structure and that in [6] is the method used to couple the two symmetrical halves of the network. In Fig. 3, the two halves are coupled via their common broadwall in the x–z plane. Any of the diagonal elements of the even-mode coupling matrix can be realized either by a circular hole in the center of the common wall between a top and a bottom cavity (electric field coupling) or by a narrow slot at the edge of the cavity (magnetic field coupling). The couplings produced by these two methods have opposite signs; hence, any arbitrary pattern of signs of the symmetrical canonical form can be realized. The canonical form realization has been experimentally verified by using a sixth-order elliptic response. Measured and computed results for this filter, together with the coupling matrix, are shown in Fig. 4.

Although the structure of Fig. 3 is quite general, a simpler mechanical structure can be obtained by using the dual-mode square-waveguide cavity geometry [3] shown in Fig. 5. In general, the canonical form of the even-mode coupling matrix may not be realizable in the square-waveguide dual-mode structure, since in this structure the signs of diagonal elements of the even-mode coupling matrix are fixed *a priori* according to the rule

$$\begin{aligned} \operatorname{sgn}[M_{e_{ii}}] &= \operatorname{sgn}[M_{e11}], && i = 4k,\, 4k+1 \\ &= -\operatorname{sgn}[M_{e11}], && i = 4k-2,\, 4k-1 \end{aligned} \tag{5}$$

where $k = 1,2,\cdots,m/2$.

This rule of signs can be deduced from simple consideration of the field configurations of orthogonal TE_{101} modes in the square-waveguide cavities at the positions of coupling slots and screws. However, in addition to the tridiagonal elements of M_e, the elements $M_{i,i+3}$, $i = 2k-1$, $k = 1,2,\cdots,m$, are nonzero; consequently, a

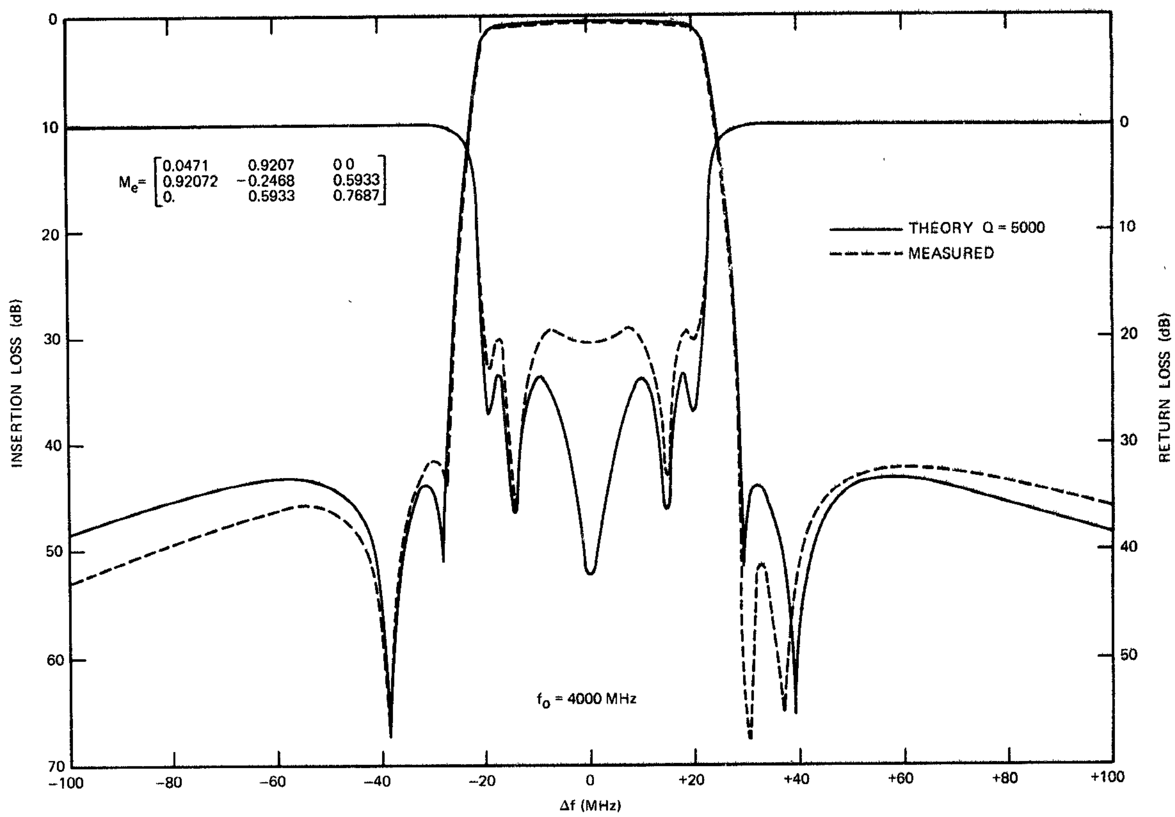

Fig. 4. Measured and computed response of sixth-order canonical elliptic waveguide filter.

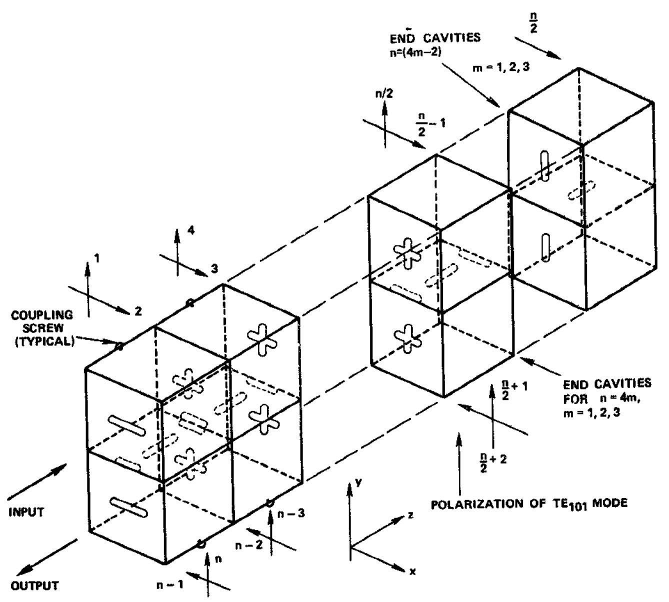

Fig. 5. Square-waveguide dual-mode cavity structure. Arrows indicate electric field direction.

large (possibly infinite) number of equivalent (orthogonally similar) even-mode coupling matrices that possess the desired diagonal pattern of signs may be generated. Methods of obtaining some of these solutions have been considered in [8]. Because there are a large number of solutions, an added advantage of the orthogonal-mode realization is that the designer can choose those solutions that are less sensitive to parameter changes and/or those element values that are more practically realizable.

TRANSFER FUNCTION AND FILTER REALIZATION

The canonical single-mode form of Fig. 3 and the orthogonal dual-mode square-waveguide structure of Fig. 5 can realize any arbitrary transfer function of order $n = 2m$ if it satisfies the conditions given in the last section and is realizable by a symmetrical network. Known types of waveguide filter response that can be realized in these structures are the following.

1) Butterworth or Chebyshev filters with nonoptimum amplitude and group delay:

$$P(S) = 1 \qquad c = 1. \tag{6}$$

2) Filters with optimum elliptic Cauer-type amplitude responses and nonoptimum group delay:

$$P(S) = \prod_{i=1}^{k} (S^2 + S_i^2), \qquad k \leq m - 1 \tag{7}$$

where the S_i are real.

3) Filters with linear phase [6]. In this case, $P(S)$ is a polynomial of degree $n - 2$ with no purely imaginary zeros; i.e., these filters possess optimum group delay and a monotonic stopband amplitude response.

In cases a and b, the transfer functions are minimum-phase-type transfer functions; hence, their amplitude and phase (group delay) characteristics are uniquely related by the Hilbert transform [10]. Since in both cases the functions are specified purely on the basis of their amplitude characteristics, the resulting group delay is by no means optimum. On the other hand, case c has a monotonic stopband amplitude response and hence is known to be nonoptimum. Therefore, to demonstrate the full potential of the general waveguide filter structures, a transfer function that has optimum-amplitude properties and flat group delay has been chosen for realization. Numerical methods have been used to compute such a transfer function. This paper does not treat the approximation problem, but instead concentrates on the physical waveguide realization.

The nonminimum-phase low-pass transfer function that has been chosen for realization [7] is

$$t(S) = \frac{cP(S)}{Q(S)} \tag{8}$$

where $c = 0.021974270$, and

$$\begin{aligned} P(S) = {} & S^{10} + 3.001410013S^8 - 1.705287093S^6 \\ & - 0.268073888S^4 + 0.602550210S^2 \\ & - 0.302240939 \end{aligned} \tag{9}$$

$$\begin{aligned} Q(S) = {} & S^{12} + 2.60284S^{11} + 6.25145S^{10} + 9.483377S^9 \\ & + 12.28017S^8 + 12.19458S^7 + 10.08583S^6 \\ & + 6.60655S^5 + 3.46316S^4 + 1.38002S^3 \\ & + 0.39859S^2 + 0.07391S + 0.00665. \end{aligned} \tag{10}$$

The computed insertion loss, return loss, and group delay response of a normalized 1-percent-bandwidth filter having this low-pass transfer function are shown in Fig. 6. The response has a high amplitude selectivity and its group delay is flat (within less than ±1-ns variation) over 80 percent of the bandwidth.

The synthesis method described in [8] can be used to evaluate a number of solutions for the even-mode coupling matrix. The canonical form, realizable in the single-mode waveguide structure shown in Fig. 3, is

$$\begin{bmatrix} -0.00844 & 0.95606 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.95606 & 0.00251 & 0.62644 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.62644 & 0.03230 & 0.56365 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.56365 & 0.02153 & 0.54058 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.54058 & -0.18774 & -0.51596 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.51596 & -0.39454 \end{bmatrix} \tag{11}$$

while a solution applicable to the square-waveguide dual-mode geometry shown in Fig. 5 is

$$\begin{bmatrix} -0.00844 & 0.89113 & 0.0 & 0.34632 & 0.0 & 0.0 \\ 0.89113 & 0.07692 & 0.45077 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.45077 & 0.27581 & 0.48101 & 0.0 & 0.3222 \\ 0.34632 & 0.0 & 0.44101 & -0.49013 & 0.13701 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.13701 & -0.9038 & 0.32458 \\ 0.0 & 0.0 & 0.3222 & 0.0 & 0.32458 & 0.51527 \end{bmatrix} \tag{12}$$

This solution was chosen for practical realization since the dual-mode geometry represents the most simple mechanical structure.

It is interesting to note that if rows and columns 4 and 6 of the canonical solution are interchanged, then the diagonal signs will satisfy the dual-mode geometry condition given by (5). However, this is not a very practical solution since M_{14} and M_{34} are 0. The canonical solution is therefore much more applicable to the single-mode structure of Fig. 3.

WAVEGUIDE DESIGN AND EXPERIMENTAL RESULTS

The design of the square-waveguide structure, which requires determination of the cavity lengths and coupling slot sizes, follows closely the methods outlined in the literature [11], [12]. A 40-MHz-bandwidth filter centered at 4 GHz was constructed and is shown in Fig. 7.

Tuning of the filter was accomplished by extending Dishal's [13] procedure to both transmission and return losses. First, cavities 1, 2, 11, and 12 were tuned with

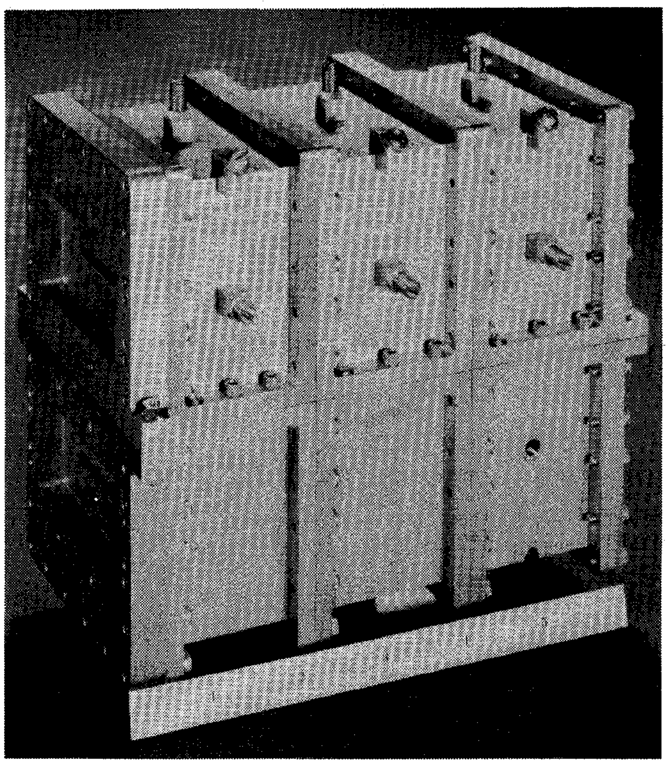

Fig. 6. Experimental filter.

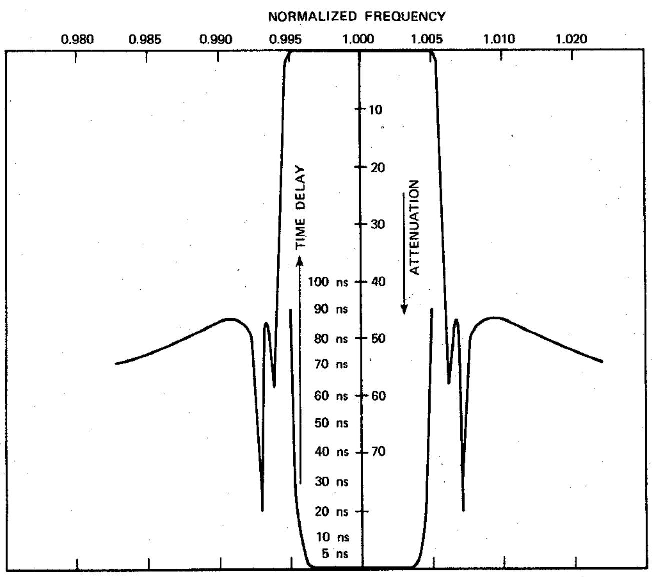

Fig. 7. Normalized equalized filter response for 1-percent bandwidth.

all the other cavities short circuited; then successive cavities were brought in until the total filter response shown in Fig. 8 was obtained.

The in-band insertion loss and group delay are shown in Fig. 9. Theory and experiment are in excellent agreement except for the return loss, which indicates coupling errors of ±2.5 percent. The average realized Q is 7500, since only parts of the structure are brazed. A fully brazed filter would yield a Q of at least 10 000 and a center frequency loss of approximately 0.9 dB.

DISCUSSION AND CONCLUSIONS

The 40-MHz 12-pole nonminimum-phase filter centered at 4 GHz demonstrated a significant improvement in the utilization efficiency of the 12 electrical cavities when compared to other known realizations. This is illustrated

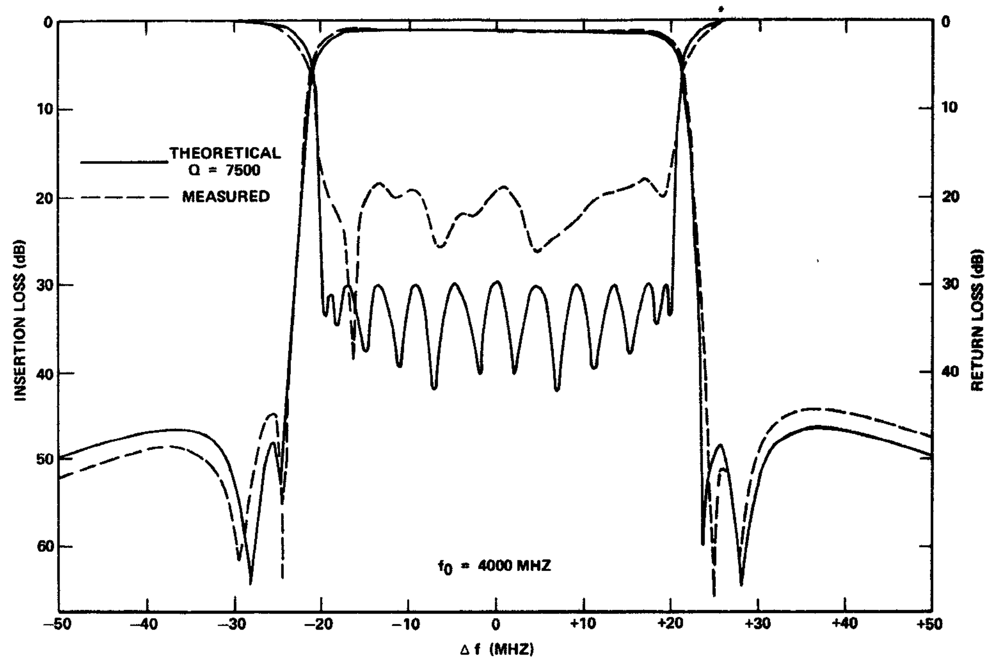

Fig. 8. Transmission and return loss responses of complete filter.

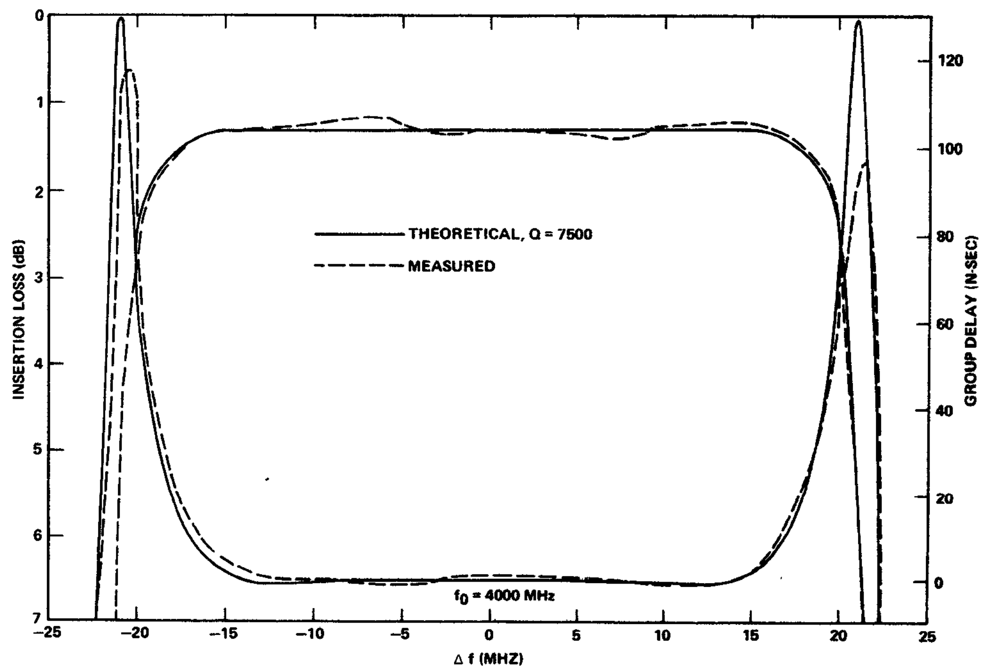

Fig. 9. In-band amplitude and group delay.

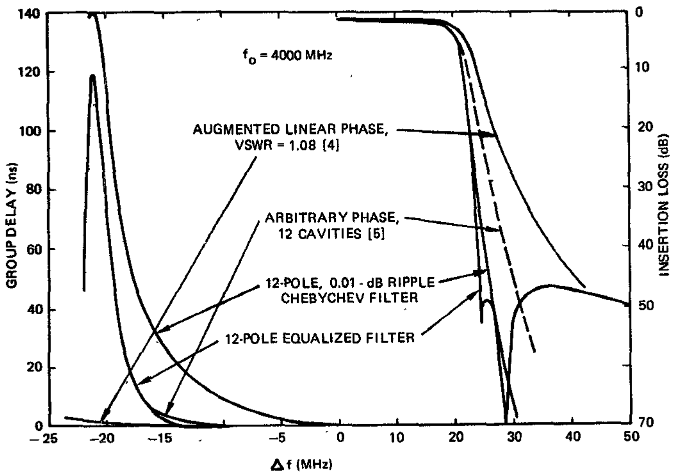

Fig. 10. Comparison of filter responses.

in Fig. 10, where a comparison is made with three other types of filters.

First, the 12-pole Chebyshev response is less selective and, for a time-delay specification of 5 ns, the usable bandwidth is only 37.5 percent. On the other hand, the 12-pole augmented linear phase polynomial exhibits excellent time-delay properties, but the amplitude selectivity is poor.

A better compromise between these two responses is provided by the arbitrary-phase polynomial transfer function realizable by a structure possessing all positive couplings. In this case, a useful tradeoff between amplitude selectivity and time delay is achieved. Such a response represents the best known waveguide realization that has previously been employed. However, a comparison with the nonminimum-phase filter described in this paper indicates that more stringent specifications can be met by a waveguide structure having the same number of cavities.

ACKNOWLEDGMENT

The authors wish to thank L. Pollack, A. Berman, and W. Getsinger for their encouragement in developing the types of filters described, and R. Kessler for arranging for the construction of the experimental filter.

REFERENCES

[1] S. B. Cohn, "Direct-coupled-resonator filters," *Proc. IRE*, vol. 45, pp. 187–196, Feb. 1957.
[2] T. A. Abele and H. Wang, "An adjustable narrow band microwave delay equalizer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 566–574, Oct. 1967.
[3] A. E. Atia and A. E. Williams, "Narrow-bandpass waveguide filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 258–265, Apr. 1972.
[4] J. D. Rhodes, "A low-pass prototype network for microwave linear phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 290–301, June 1970.
[5] ——, "Filters approximating ideal amplitude and arbitrary phase characteristics," *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 120–124, Mar. 1973.
[6] ——, "The generalized direct-coupled cavity linear phase filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 301–308, June 1970.
[7] K. Wittman, G. Pfitzenmaier, and F. Kunemund, "Dimensionierung Reflexionsfaktor und Laufzeitgeebneter Versteilaster Filter mit Uberbouckungen," *Frequenz*, vol. 24, pp. 307–312, Oct. 1970.
[8] A. E. Atia and A. E. Williams, "New types of waveguide bandpass filters for satellite transponders," *COMSAT Tech. Rev.*, vol. 1, pp. 21–43, Fall 1971.
[9] A. E. Williams and A. E. Atia, "Two port synthesis of narrow band coupled cavities," in *Proc. Int. Filter Symp.* (Santa Monica, Calif.), Apr. 1972, pp. 67–70.
[10] E. Guillman, *Synthesis of Linear Networks*. New York: Wiley, 1957.
[11] Matthaei *et al.*, *Microwave Filters, Impedance Matching Networks and Coupling Structures*. New York: McGraw-Hill, 1965.
[12] S. B. Cohn, "Microwave coupling by large apertures," *Proc. IRE*, vol. 40, pp. 696–699, June 1952.
[13] M. Dishal, "Alignment and adjustment of synchronously tuned multiple-resonant-circuit filters," *Proc. IRE*, vol. 39, pp. 1448–1455, Nov. 1951.